Ultra Chemistry Vol. 12(2), 52-54 (2016).
www.journalofchemistry.org
ISSN 0973-3450 (Print)
2319-8036 (Online)

A Fixed Point Theorem in Rational Form on Compact Metric Space

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(Acceptance Date 10th April, 2016, Online Publication date 2 Sep. 2016)

Abstract

In this paper, we prove a fixed point theorem for self mappings satisfying a new contractive type condition in a compact metric space. Fixed point theorem is useful in mathematical model for an adiabatic tubler chemical reactor which processes an irreversible exothermic chemical reaction. It is also useful in chemical similarity regarding substances as elements of a compact metric space.

Key words: Fixed point, compact metric space, Fixed point theorem. Chemical similarity and chemical reactor.

Introduction

In 1922, the Polish mathematician Stefan Banach proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. His result is called Banachs fixed point theorem. This result provides a technique for solving variety of applied problems in mathematical science and engineering. In the wide range of mathematical problems the existence of a solution is equivalent to the existence of a fixed point for a suitable map. The existence of a fixed point is therefore of paramount importance in several area of mathematics and chemistry. When fixed point theorem comes to medical science, the Banach fixed point theorem will be often present when some relevant chemical process is modelled by equations. These problems arise in chemical reactor theory and depending on the reaction rate and may exhibit multiple solutions corresponding to multiple steady states in the chemical reactions. Many authors like Edalstien[1], Kanon[3], Soni[4] and Sahu[7] have extended, generalized and improved Banach fixed point theorem in different ways.

In this paper, we extend the work of Sahu for self mapping satisfying a new contractive type condition in a compact metric space.

Throughout this paper the compact metric space (X, d) is denoted by X.

Main Result :

Theorem: Let F be a continuousself mapping defined on a compact metric space X with

$$d(Fx, Fy) \le k_1 \frac{d(x,Fx)d(y,Fx)+d(y,Fy)d(x,Fy)}{d(x,Fx)+d(y,Fx)+d(y,Fy)+d(x,Fy)} +$$

$$k_2 \frac{d(x,Fx)d(y,Fy)+d(x,Fy)d(y,Fx)}{d(x,Fx)+d(y,Fy)+d(x,Fy)+d(y,Fx)} +$$

$$k_3 \frac{d(x,Fx)d(x,Fy)+d(y,Fx)d(y,Fy)}{d(x,Fx)+d(x,Fy)+d(y,Fx)+d(y,Fy)} +$$

$$k_4 \frac{d(x,Fx)d(x,Fx)+d(x,Fy)d(y,Fx)}{d(x,Fx)+d(x,Fx)+d(x,Fy)+d(y,Fx)}$$

$$+ k_5 [d(x, Fx) + d(y, Fx)] + k_6\, d(x, y) \qquad (1)$$

$\forall x, y \in X$, $x \neq y$ and $k_1 + k_2 + k_3 + k_4 + 2k_5 + 2k_6 < 2$, then F has a fixed point. Further when $k_2 + k_4 + 2k_5 + 2k_6 < 2$. Then F has a unique fixed point.

Proof: First we define a function G on X as follows, Gx = d(x, Fx).
Since d and Fare continuous on X, therefore G is also continuous on X.
Since X is compact, there exist a point $p \in X$ such that

$$Gp = \inf \{Gx : x \in X\} \qquad (2)$$

If Gp = 0, then d(p, Fp) = 0 i.e. p = Fp
So G(Fp) = d(Fp, F(Fp))

$$\text{Now } d(Fp,F(Fp)) \le k_1 \frac{d(p,Fp)d(Fp,Fp)+d(Fp,F(Fp))d(p,F(Fp))}{d(p,Fp)+d(Fp,Fp)+d(Fp,F(Fp))+d(p,F(Fp))}$$

$$+k_2 \frac{d(p,Fp)d(Fp,F(Fp))+d(p,F(Fp))d(Fp,Fp)}{d(p,Fp)+d(Fp,F(Fp))+d(p,F(Fp))+d(Fp,Fp)}$$

$$+k_3 \frac{d(p,Fp)d(p,F(Fp))+d(Fp,Fp)d(Fp,F(Fp))}{d(p,Fp)+d(p,F(Fp))+d(Fp,Fp))+d(Fp,F(Fp))}$$

$$+k_4 \frac{d(p,Fp)d(p,Fp)+d(p,F(Fp))d(Fp,Fp)}{d(p,Fp)+d(p,Fp)+d(p,F(Fp))+d(Fp,Fp)}$$

$$+ k_5[d(p, Fp) + d(Fp, Fp)] + k_6\, d(p, Fp).$$

$$\text{Thus } d(Fp, F(Fp)) \le \frac{k_1}{2} d(Fp, F(Fp)) + \frac{k_2}{2} d(Fp, F(Fp)) + \frac{k_3}{2} d(p, Fp) + \frac{k_4}{2} d(p, Fp) + k_5\, d(p, Fp) + k_6\, d(p, Fp).$$

$$\text{i.e. } d(Fp, F(Fp)) \le \frac{\frac{k_3}{2}+\frac{k_4}{2}+k_5+k_6}{1-(\frac{k_1}{2}+\frac{k_2}{2})} d(p, Fp)$$

i.e. $d(Fp, F(Fp)) \le k\, d(p, Fp)$.

$$\text{where } k = \frac{\frac{k_3}{2}+\frac{k_4}{2}+k_5+k_6}{1-(\frac{k_1}{2}+\frac{k_2}{2})} \text{ and } k < 1, \text{ Since}$$

$k_1 + k_2 + k_3 + k_4 + 2k_5 + 2k_6 < 2$.

ThusG(Fp)<Gp, which is contradiction to the condition (2).
So Fp = p. Consequentlyp is a fixed point of F in X.
Now we show that p is unique. For suppose q be other fixed point such thatFq = q,

$$\text{Then } d(p, q) = d(Fp, Fq) \le k_1 \frac{d(p,Fp)d(q,Fp)+d(q,Fq))d(p,Fq)}{d(p,Fp)+d(q,Fp)+d(q,Fq))+d(p,Fq))}$$

$$+k_2 \frac{d(p,Fp)d(q,Fq)+d(p,Fq)d(q,Fp)}{d(p,Fp)+d(q,Fq))+d(p,Fq))+d(q,Fp)}$$

$$+k_3 \frac{d(p,Fp)d(p,Fq)+d(q,Fp)d(q,Fq)}{d(p,Fp)+d(p,Fq)+d(q,Fp))+d(q,Fq)}$$

$$+k_4 \frac{d(p,Fp)d(p,Fp)+d(p,Fq)d(q,Fp)}{d(p,Fp)+d(p,Fp)+d(p,Fq)+d(q,Fp)}$$

$$+ k_5[d(p, Fp) + d(q, Fp)] + k_6\, d(p, q).$$

Thus $d(p, q) < (\frac{k_2}{2} + \frac{k_4}{2} + k_5 + k_6)\, d(p, q)$.

Which is a contradiction because $k_2 + k_4 + 2k_5 + 2k_6 < 2$.
Thus d(p, q) = 0 i. e. p = q.
Hence F has a unique fixed point.

Remark:

(i) If $k_1 = k_2 = k_3 = k_4 = k_5 = 0$ and $k_6 = 1$then the theorem reduce to Edelstien[1].

(ii) If $k_4 = 0$ then the theorem reduce to Sahu[7].

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